The ^{132}Sn giant dipole resonance as a constraint on nuclear matter properties

B. Roach[1,2], G. Bonasera[2], and S. Shlomo[2]

1. Department of Physics, University of Notre Dame, Notre Dame, IN 46556, USA
2. Cyclotron Institute, Texas A&M University, College Station, TX 77843, USA

CYCLOTRON INSTITUTE
TEXAS A&M UNIVERSITY

Introduction

Nuclear giant resonances provide a sensitive method for constraining the properties of nuclear matter (NM) - many of which have large uncertainties - and thereby improve the nuclear energy-density functional. In this work, self-consistent Hartree-Fock random-phase approximation (HF-RPA) theory was employed to calculate the strength function and energy of the isovector giant dipole resonance (IVGDR) in the doubly-magic ^{132}Sn nucleus. Several (17) commonly-used Skyrme-type interactions were employed. The correlations between the IVGDR centroid energy and each nuclear matter property were explored, as were correlations between the nuclear matter properties and the ^{132}Sn neutron skin thickness $r_n - r_p$. Experimental data for the IVGDR centroid energy was used to constrain the symmetry energy density at saturation J and the first and second density derivatives L and K_{sym}, respectively, of the NM symmetry energy. Further investigation, particularly of neutron-rich nuclides far from stability, will be needed to extend the nuclear energy-density functional to the extremes of density and neutron abundance found in neutron stars and astrophysical nucleosynthesis environments.

Nuclear giant resonances

The phenomenon of nuclear giant resonance, in which the reaction cross-section σ is strongly peaked at a specific energy, was first ascribed to collective motion of nucleons by Goldhaber and Teller [1]. Giant resonances are classified according to their multipolarity L and isospin exchange ΔT, representing the shape and phase of the neutron-proton oscillations. In the isovector giant dipole resonance (IVGDR), the proton and neutron distributions oscillate in alternating directions, analogous to an electromagnetic dipole radiation source.

The giant resonance cross-section as a function of energy is customarily fit with a Lorentzian, peaked at $\sigma(E_R) = \sigma_R$ and with full-width at half-maximum (FWHM) 2Γ:

$$\sigma(E) = \frac{E^2\Gamma^2\sigma_R}{(E_R^2 - E^2)^2 + E^2\Gamma^2}$$

Infinite nuclear matter

Determining the equation of state of infinite nuclear matter (*i.e.*, the binding energy per nucleon as a function of density) is of primary importance in many fields of nuclear physics and astrophysics. Many properties of infinite NM have been established, including nucleon saturation density ρ_0 (~0.16 fm^{-3}) and binding energy (~16 MeV/A). In the general (asymmetric NM) case [2],

$$E(\rho_p, \rho_n) = E_0(\rho_p) + E_{sym}(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)$$

where the symmetry energy (the favorability of the neutron-proton interaction) is

$$E_{sym}(\rho) = J + L\left(\frac{\rho - \rho_0}{\rho_0}\right) + \frac{1}{18}K_{sym}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2$$

$$\begin{cases} J = E_{sym}(\rho_0) \\ L = 3\rho_0 \left.\frac{\partial E_{sym}}{\partial \rho}\right|_{\rho_0} \\ K_{sym} = 9\rho_0 \left.\frac{\partial^2 E_{sym}}{\partial \rho^2}\right|_{\rho_0} \end{cases}$$

The quantities J, L, and K_{sym} are the focus of the present work.

Hartree-Fock theory

Hartree-Fock (HF) theory addresses the complexity of the A-nucleon interacting system by assuming that each nucleon moves in a mean-field central potential U created by the others, leaving a residual term:

$$H = \sum_{\substack{i,j=1 \\ i\neq j}}^{A}\left[\frac{\vec{p}_i^2}{2m_i} + V(\vec{r}_i, \vec{r}_j)\right] = \sum_{i=1}^{A}\left[\frac{\vec{p}_i^2}{2m_i} + U(\vec{r}_i)\right]_{mf} + H_{res}$$

Each nucleon must satisfy the single-particle Schrodinger equation with the mean-field Hamiltonian described above. Since nucleons are fermions, their wavefunctions are antisymmetric, so the overall nuclear wavefunction is a Slater determinant of the single-particle solutions:

$$\Phi = \frac{1}{\sqrt{A!}}\begin{vmatrix} \phi_1(\vec{r}_1, \sigma_1, \tau_1) & \phi_2(\vec{r}_1, \sigma_1, \tau_1) & \dots & \phi_A(\vec{r}_1, \sigma_1, \tau_1) \\ \phi_1(\vec{r}_2, \sigma_2, \tau_2) & \phi_2(\vec{r}_2, \sigma_2, \tau_2) & \dots & \phi_A(\vec{r}_2, \sigma_2, \tau_2) \\ \vdots & \ddots & & \vdots \\ \phi_1(\vec{r}_A, \sigma_A, \tau_A) & \phi_2(\vec{r}_A, \sigma_A, \tau_A) & \dots & \phi_A(\vec{r}_A, \sigma_A, \tau_A) \end{vmatrix}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the spin and isospin quantum numbers, respectively. One may then invoke a variational principle to minimize the expectation value of the ground-state energy $\langle\phi|H|\phi\rangle$, introduce a perturbation to the original single-particle wavefunctions, and obtain the HF equations

$$-\frac{\hbar^2}{2m}\nabla_r^2\phi_{\alpha_j}(\vec{r}) + U_H(\vec{r})\phi_{\alpha_j}(\vec{r}) - \int U_F(\vec{r}, \vec{r}')\phi_{\alpha_j}(\vec{r}')d^3\vec{r}' = e_{\alpha_j}\phi_{\alpha_j}(\vec{r}) \qquad j = 1, \dots, A$$

where the second and third terms are contributions from the direct term (local potential) and exchange term (nonlocal potential), respectively:

$$U_H(\vec{r}) = \int \rho(\vec{r}')V(|\vec{r} - \vec{r}'|)d\vec{r}' \qquad U_F(\vec{r}, \vec{r}') = \sum_{i=1}^{A}\phi^*_{\alpha_i}(\vec{r}')V(|\vec{r} - \vec{r}'|)\phi_{\alpha_i}(\vec{r})$$

Random-phase approximation

Macroscopically, the giant resonance is described as collective motion in a neutron-proton "liquid drop"; microscopically, it is a coherent set of particle-hole excitations. A successful microscopic approach is the random-phase approximation (RPA), which provides an orthonormal basis for the nuclear oscillations. The details of the RPA formalism and derivation are deferred to [3]. The main results for the present work are the strength function $S(E)$ and its k^{th} moment m_k corresponding to the scattering operator F over the n RPA states [2]:

$$S(E) = \sum_n |\langle 0|F|n\rangle|^2\,\delta(E - E_n) \qquad m_k = \int nE^k S(E)dE$$

The single-particle scattering operator $\mathbf{F_L}$ for the isovector L-multipole is

$$\hat{F}_L = \frac{Z}{A}\sum_n f(r_n)Y_{L0}(n) - \frac{N}{A}\sum_p f(r_p)Y_{L0}(p)$$

and the IVGDR radial dependence is $f(r) = r$. The centroid energy is $E_{cen} = m_1/m_0$.

Skyrme interaction

We adopt the standard form [4,5] of the Skyrme effective nucleon-nucleon interaction

$$V_{ij}^{NN} = t_0(1 + x_0P_{ij}^{\sigma})\delta(\vec{r}_i - \vec{r}_j) + \tfrac{1}{2}t_1(1 + x_1P_{ij}^{\sigma})\left[\overleftarrow{k}_{ij}^2\delta(\vec{r}_i - \vec{r}_j) + \delta(\vec{r}_i - \vec{r}_j)\vec{k}_{ij}^2\right] + t_2(1 + x_2P_{ij}^{\sigma})\overleftarrow{k}_{ij}^2\delta(\vec{r}_i - \vec{r}_j)\vec{k}_{ij} + \tfrac{1}{6}t_3(1 + x_3P_{ij}^{\sigma})\rho^{\alpha}\left(\frac{\vec{r}_i + \vec{r}_j}{2}\right)\delta(\vec{r}_i - \vec{r}_j) + iW_0\overleftarrow{k}_{ij}\delta(\vec{r}_i - \vec{r}_j)(\sigma_i + \sigma_j)\vec{k}_{ij}$$

with 10 parameters x_i, t_i, W_0 and α. The Skyrme interaction has a corresponding energy-density functional $E = \int H(\vec{r})d\vec{r}$, where the Hamiltonian includes kinetic, Coulomb, and Skyrme interaction contributions. The full treatment of the Skyrme Hamiltonian is deferred to [5], though some general notes are relevant. The Skyrme Hamitonian terms include zero-range, density-dependent, spin-orbit, effective-mass, finite-range, and spin-gradient contributions, which can be expressed in terms of the Skyrme interaction parameters and the (gradient of) nuclear densities. The properties of nuclear matter can then be calculated from the Skyrme parameters. It is also common to invert the relationship, and use (empirical) nuclear matter properties to derive a new Skyrme parameterization, *e.g.* via a simulated-annealing approach [6].

Calculations and results

All calculations were carried out using the self-consistent Hartree-Fock random-phase approximation (HF-RPA) theory, described in [2,7]. Seventeen commonly-used Skyrme interactions were implemented; an extensive list can be found in [8]. The values of the NM properties were calculated for each interaction, and the ranges of these values for the Skyrme interactions used are contained in the table below, in the form (x_{min}, x_{max}):

ρ_0	E/A	J	L	K_{sym}	m*/m	κ
0.156, 0.175	16.00, 16.34	26.7, 37.4	-29.4, 129.3	-401.4, 159.6	0.58, 1.00	0.23, 0.71

In **Figure I**, the correlations between the IVGDR centroid energy E_{cen}, and the symmetry energy at saturation density, J, and the first and second density derivatives L and K_{sym}, respectively, of the NM symmetry energy, become apparent. The dashed vertical lines denote the experimental uncertainty from [9]. In spite of the correlations, the spread of the data within the experimental range is too large to suggest a tighter band of uncertainty for any of the symmetry energy quantities.

Figure II shows the correlations between the ^{132}Sn neutron skin thicnkess $r_n - r_p$ and the same symmetry energy parameters. A particularly strong correlation (0.85) was observed between the skin thickness and L. Similarly to **Figure I**, the data are spread too widely to suggest a unique relation between $r_n - r_p$ and the aforementioned quantities.

Figure III (top right) demonstrates the weak correlation between the centroid energy and the effective mass m^*/m. Since the symmetry energy and its derivatives are moderately well-correlated to the centroid energy, it follows that the effective mass (which theory predicts to vary inversely with the centroid energy) is less-correlated.

Figure IV (top right) shows the relationship between the neutron skin thickness $r_n - r_p$ and the electric dipole polarizability α_D. Unlike the very strong correlation (0.98) described in [10] regarding ^{208}Pb, the present results for ^{132}Sn indicate a much weaker correlation (0.40). Additionally, a measurement of α_D would not uniquely determine a value of the neutron skin thickness, according to the present calculations.

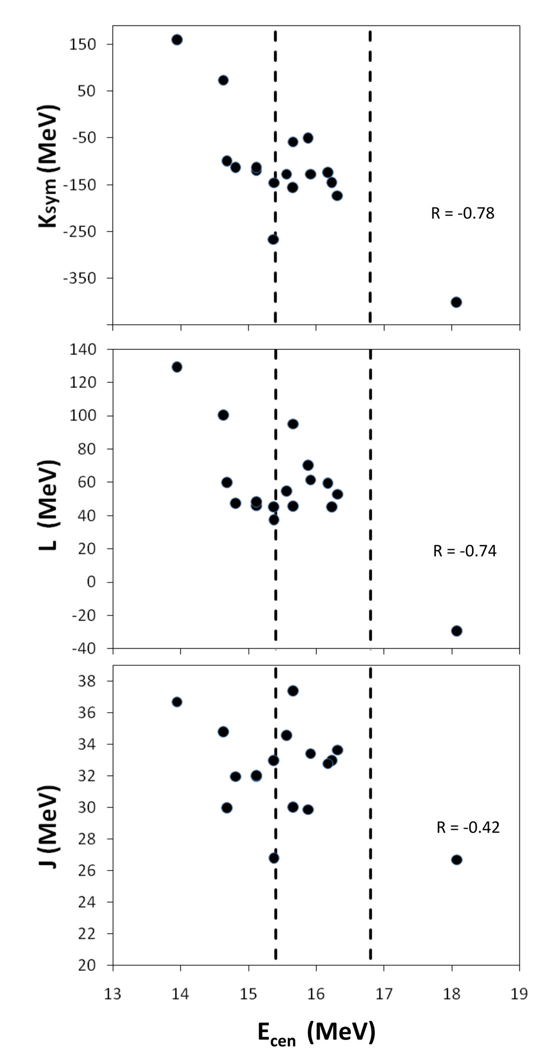

Figure I: The symmetry energy quantities J, L, and K_{sym} and plotted against the centroid energy E of the ^{132}Sn IVGDR. The dashed vertical lines denote the experimental uncertainty from [9]. Pearson correlation coefficients R are also indicated.

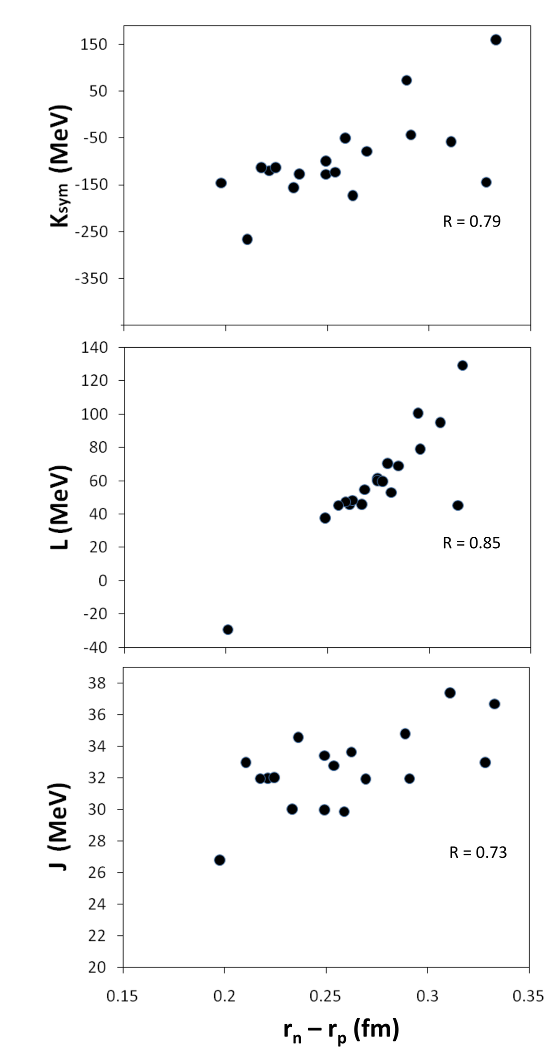

Figure II: Same as **Figure I**, but with the neutron skin thickness $r_n - r_p$.

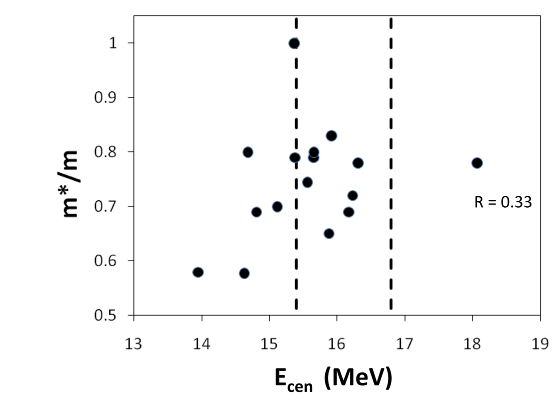

Figure III: Same as **Figure I**, but with the effective mass m^*/m.

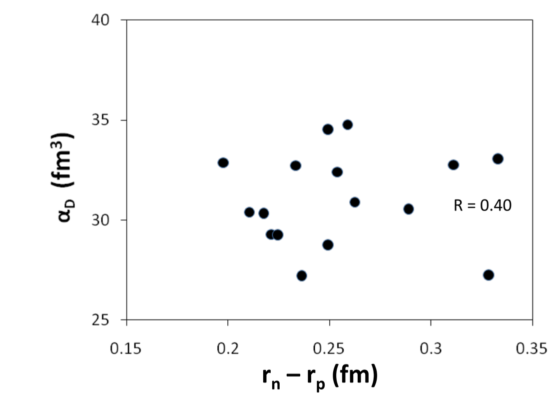

Figure IV: Same as **Figure II**, but with the electric dipole polarizability α_D.

Conclusions and outlook

The IVGDR centroid energy E_{cen} of the doubly-magic, neutron-rich nucleus ^{132}Sn has been calculated using the Hartree-Fock random-phase approximation theory, for a variety of commonly-used Skyrme interactions. The correlations between E_{cen} and properties of infinite nuclear matter, particularly the symmetry energy and its first and second derivatives, and the electric dipole polarizability α_D, have been explored. In spite of the correlations, the calculated values are too scattered to allow for tightened constraints on the values of the nuclear matter properties, though the calculated values are consistent with those determined experimentally. In particular, the observed correlation of α_D with the neutron skin thickness $r_n - r_p$ was weaker than for the ^{208}Pb nucleus. Performing similar calculations with additional Skyrme-type interactions will be critical for determining the accuracy of the observed correlations. Further study of exotic, neutron-rich nuclei will be necessary to better constrain the values of these properties. The behavior of the symmetry energy parameters at high density and neutron asymmetry is particularly relevant to the structure of neutron stars, and governs the late stages of stellar evolution and processes of explosive nucleosynthesis.

References

[1] M. Goldhaber and E. Teller. PR **74** (1948)

[2] M. R. Anders, S. Shlomo, Tapas Sil, D. H. Youngblood, Y.-W. Lui, and Krishichayan. PRC **87**, 024303 (2013).

[3] J. Suhonen. *From Nucleons to Nucleus.* Springer: New York (2007).

[4] T. H. R. Skyrme. Nuclear Physics **9** (1959).

[5] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, R. Schaeffer. Nuclear Physics A **627** (1997).

[6] B. K. Agrawal, S. Shlomo, and V. Kim Au. PRC **72**, 014310 (2005).

[7] G. Colò, L. Cao, N. Van Giai, and L. Capelli. Computer Phys. Communications **184** (2013).

[8] M. Dutra, O. Lourenço, J. S. Sá Martins, A. Delfino, J. R. Stone, and P. D. Stevenson. PRC **85**, 035201 (2012).

[9] P. Adrich, A. Klimkiewicz, M. Fallot, K. Boretzky, T. Aurmann, D. Cortina-Gil, U. Datta Pramanik, Th. W. Elze, H. Emling, H. Geissel, M. Hellström, K. L. Jones, J. V. Kratz, R. Kulessa, Y. Leifels, C. Nocifero, R. Palit, H. Simon, G. Surówka, K. Sümmerer, and W. Waluś. PRL **92**, 132501 (2005).

[10] P.-G. Reinhard and W. Nazarewicz. PRC **81**, 051303(R) (2010).

Acknowledgements

The support of the Texas A&M Cyclotron Institute, in particular Prof. Yennello and the other coordinators of the REU program, is gratefully acknowledged. B.R. would also like to thank M. Anders for providing data on the Skyrme interactions and nuclear matter properties. This research was supported by the National Science Foundation, Grant 1263281.